Lecture 10

Relationships between Categorical Variables

Thought Question 1:

Students in a statistics class were asked whether they preferred an **in-class or a take-home final exam** and were then categorized as to **whether they had received an A** on the midterm.

Of the 25 A students, 10 preferred a take-home exam, whereas of the 50 non-A students, 30 preferred a take-home exam.

How would you display these data in a table?

Thought Question 2:

Suppose a news article claimed that **drinking coffee doubled your risk of developing a certain disease**. Assume the statistic was based on legitimate, well-conducted research.

What **additional information** would you want about the risk before deciding whether to quit drinking coffee?

(*Hint:* Does this statistic provide any information on your actual risk?)

Thought Question 3:

A study classified pregnant women according to whether they smoked and whether they were able to get pregnant during the first cycle in which they tried to do so. What do you think is the **question of interest**? Attempt to answer it. Here are the results:

	Pregnancy Occurred After		
	First Cycle	**Two or More Cycles**	**Total**
Smoker	29	71	100
Nonsmoker	198	288	486
Total	227	359	586

Thought Question 4:

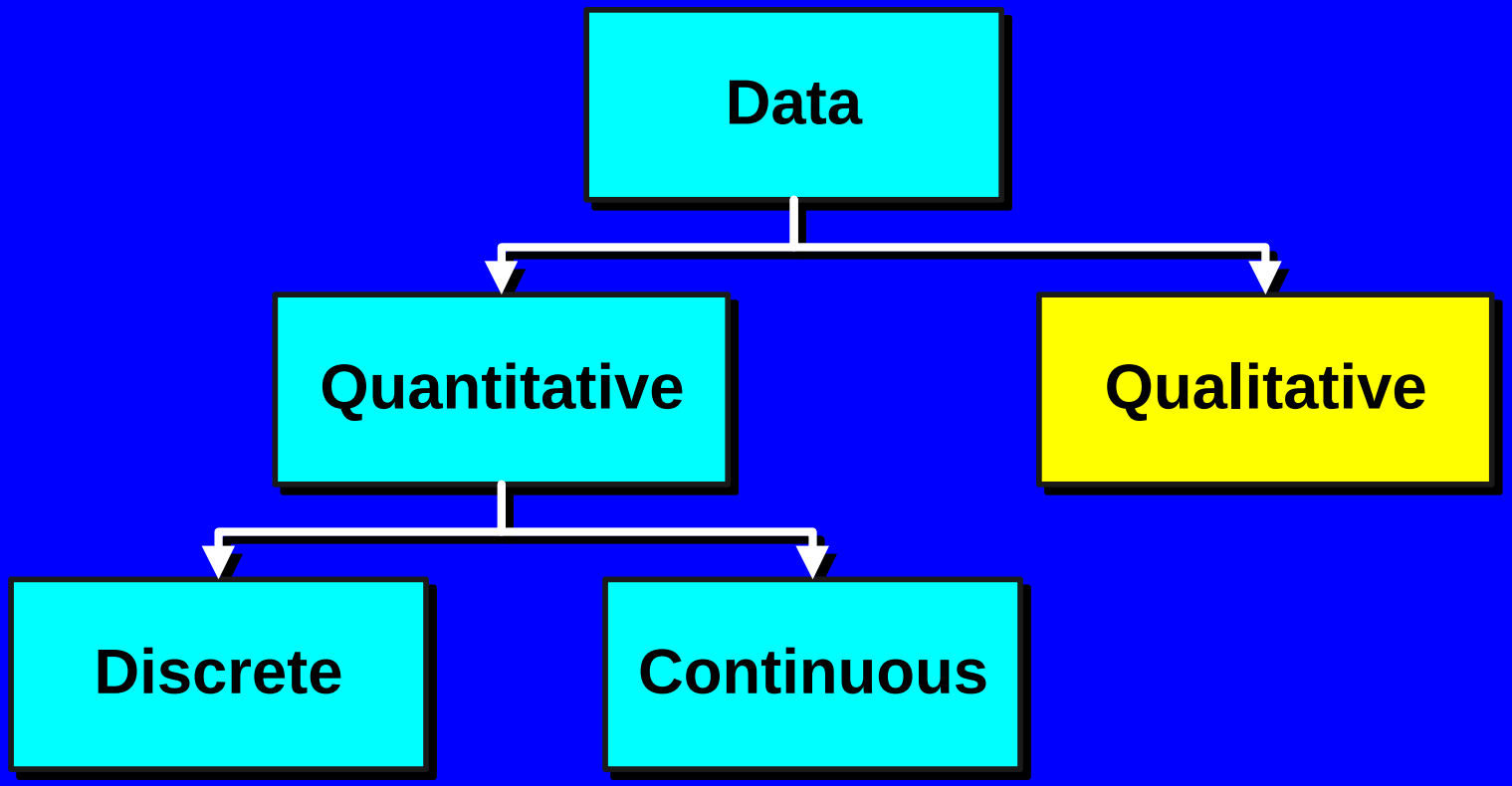
A recent study estimated that the "**relative risk**" of a woman developing lung cancer if she smoked was 27.9.

What do you think is meant by the term *relative risk*?

Qualitative Data

- 1. Qualitative Random Variables Yield Responses That Classify
  - Example: Gender (Male, Female)
- 2. Measurement Reflects # in Category
- 3. Nominal or Ordinal Scale
- 4. Examples
  - Do You Own Savings Bonds?
  - Do You Live On-Campus or Off-Campus?

Data Types



12.1 Displaying Relationships Between Categorical Variables: Contingency Tables

- Count the number of individuals who fall into each combination of categories.
- Present counts in table = **contingency table**.
- Each row and column combination = *cell*.
- Row = *explanatory* variable.
- Column = *response* variable.

A Contingency Table is a simple way to summarize data resulting from the measurement of two categorical variables.

To create a Contingency Table, simply count the number of individuals who fall into each combination of categories.

Example: An investigator was interested in determining whether or not there is a relationship between diet (high or low fat) and longevity in mice. She obtained a sample of 240 mice and randomly assigned 120 to receive a low-fat diet and 120 to receive a high-fat diet. Then she recorded how long each of the mice lived.

	Low Fat Diet	High Fat Diet	Total
Lived < 1 year	12	84	96
Lived > 1 year	108	36	144
Total	120	120	240

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Lived < 1 year	12	84	96
Lived > 1 year	108	36	144
Total	120	120	240

Proportion of mice on Low-Fat Diet who lived more than 1 year is 108/120 = 0.9

Proportion of mice on High-Fat Diet who lived more than 1 year is 36/120 = 0.3

	Low Fat Diet	High Fat Diet	Total
Lived < 1 year	12	84	96
Lived > 1 year	108	36	144
Total	120	120	240

Explanatory Variable = Diet (High or Low Fat)

Response Variable = Life span (Greater than or less than 1 year)

It is conventional to place the explanatory variable in rows, and the response variable as columns!

Addressing the Statistical Significance of a Contingency Table

•If we are working with a sample, we can't be sure that population proportions will be exactly the same as the sample proportions.

•Therefore, researchers are interested in assessing whether the differences in the observed proportions are just chance differences or represent a real difference for the population.

•A relationship is considered statistically significant if the strength of the observed relationship is stronger than what would be expected by chance. Specifically, we require such a relationship to be larger than 95% of those that would be observed just by chance.

For a 2 x 2 table, the strength of a relationship is measured by the difference in the proportions of outcomes for the two categories of the explanatory variable.

Example: The difference in proportion of rats who lived more than one year between high-fat and low-fat mice is 0.9 - 0.3 =0.6

Is this difference large enough to rule out chance?
Can we conclude that the relationships observed for the sample also holds for the population?

We need to compute the chi-squared statistic!

Example 1: Aspirin and Heart Attacks

Case Study 1.2:

Variable A = explanatory variable = aspirin or placebo
Variable B = response variable = heart attack or no heart attack

Contingency Table with explanatory as row variable, response as column variable, four cells.

	Heart Attack	No Heart Attack	Total
Aspirin	104	10,933	11,037
Placebo	189	10,845	11,034
Total	293	21,778	22,071

Conditional Percentages and Rates

Question of Interest: Do the percentages in each category of the response variable change when the explanatory variable changes?

Example 1: Find the Conditional (Row) Percentages

Aspirin Group:
Percentage who had heart attacks = 104/11,037 = 0.0094 or 0.94%

Placebo Group:
Percentage who had heart attacks = 189/11,034 = 0.0171 or 1.71%

Conditional Percentages and Rates

Rate: the number of individuals per 1000 or per 10,000 or per 100,000.

Percentage: rate per 100

Example 1: Percentage and Rate Added

	Heart Attack	No Heart Attack	Total	Heart Attacks (%)	Rate per 1000
Aspirin	104	10,933	11,037	0.94	9.4
Placebo	189	10,845	11,034	1.71	17.1
Total	293	21,778	22,071		

Example 2: Young Drivers, Gender, and Driving Under the Influence of Alcohol

Case Study 6.5: Court case challenging law that differentiated the ages at which young men and women could buy 3.2% beer.

Results of Roadside Survey for Young Drivers

	Drank Alcohol in Last 2 Hours?			
	Yes	No	Total	Percentage Who Drank
Males	77	404	481	16.0%
Females	16	122	138	11.6%
Total	93	526	619	15.0%

Percentage slightly higher for males, but difference in percentages is not ***statistically significant***.

Source: Gastwirth, 1988, p. 526.

Example 3: Ease of Pregnancy for Smokers and Nonsmokers

Retrospective Observational Study:

Variable A = explanatory variable = smoker or nonsmoker

Variable B = response variable = pregnant in first cycle or not

Time to Pregnancy for Smokers and Nonsmokers

	Pregnancy Occurred After			
	First Cycle	Two or More Cycles	Total	Percentage in First Cycle
Smoker	29	71	100	29%
Nonsmoker	198	288	486	41%
Total	227	359	586	

Much higher percentage of nonsmokers than smokers were able to get pregnant during first cycle, but we *cannot conclude* that smoking ***caused*** a delay in getting pregnant.

12.2 Relative Risk, Increased Risk, and Odds

A population contains 1000 individuals,
of which 400 carry the gene for a disease.

Equivalent ways to express this proportion:

- Forty ***percent*** (40%) of all individuals carry the gene.
- The ***proportion*** who carry the gene is 0.40.
- The ***probability*** that someone carries the gene is .40.
- The ***risk*** of carrying the gene is 0.40.
- The ***odds*** of carrying the gene are 4 to 6 (or 2 to 3, or 2/3 to 1).

Risk, Probability, and Odds

Percentage with trait =
(number with trait/total)×100%

Proportion with trait = number with trait/total

Probability of having trait = number with trait/total

Risk of having trait = number with trait/total

Odds of having trait =
(number with trait/number without trait) to 1

Baseline Risk and Relative Risk

Baseline Risk: risk without treatment or behavior

- Can be difficult to find.
- If placebo included,
  baseline risk = risk for placebo group.

Relative Risk: of outcome for two categories of explanatory variable is ratio of risks for each category.

- *Relative risk of 3*: risk of developing disease for one group is 3 times what it is for another group.
- *Relative risk of 1*: risk is same for both categories of the explanatory variable (or both groups).

Example 4: Relative Risk of Developing Breast Cancer

First Child at Age 25 or Older?	Breast Cancer	No Breast Cancer	Total
Yes	31	1597	1628
No	65	4475	4540
Total	96	6072	6168

- Risk for women having first child at 25 or older
  = 31/1628 = 0.0190
- Risk for women having first child before 25
  = 65/4540 = 0.0143
- Relative risk = 0.0190/0.0143 = 1.33

Risk of developing breast cancer is 1.33 times greater for women who had their first child at 25 or older.

Source: Pagano and Gauvreau (1988, p. 133).

Increased Risk

Increased Risk = (change in risk/baseline risk)×100%
= (relative risk – 1.0)×100%

Example 5: Increased Risk of Breast Cancer

- Change in risk = (0.0190 – 0.0143) = 0.0047
- Baseline risk = 0.0143
- Increased risk = (0.0047/0.0143) = 0.329 or 32.9%

There is a **33% increase in the chances of breast cancer** for women who have not had a child before the age of 25.

Odds Ratio

Odds Ratio: ratio of the odds of getting the disease to the odds of not getting the disease.

Example: Odds Ratio for Breast Cancer

- Odds for women having first child at age 25 or older
  = 31/1597 = 0.0194
- Odds for women having first child before age 25
  = 65/4475 = 0.0145
- Odds ratio = 0.0194/0.0145 = 1.34

Alternative formula: $\text{odds ratio} = \frac{31 \times 4475}{1597 \times 65} = 1.34$

Relative Risk and Odds Ratios in Journal Articles

Researchers often report relative risks and odds ratios ***adjusted*** to account for confounding variables.

Example:
Suppose relative risk for getting cancer for those with high-fat and low-fat diet is 1.3, adjusted for age and smoking status. =>

Relative risk applies (approx.) for two groups of individuals of same age and smoking status, where one group has high-fat diet and other has low-fat diet.

12.3 Misleading Statistics about Risk

Common ways the media misrepresent statistics about risk:

1. **The baseline risk is missing.**
2. **The time period of the risk is not identified.**
3. **The reported risk is not necessarily your risk.**

Missing Baseline Risk

"Evidence of new cancer-beer connection"
Sacramento Bee, March 8, 1984, p. A1

- Reported men who drank 500 ounces or more of beer a month (about 16 ounces a day) were ***three times more likely*** to develop cancer of the rectum than nondrinkers.
- Less concerned if chances go from 1 in 100,000 to 3 in 100,000 compared to 1 in 10 to 3 in 10.
- Need baseline risk (which was about 1 in 180) to help make a lifestyle decision.

Risk over What Time Period?

"Italian scientists report that a diet rich in animal protein and fat—cheeseburgers, french fries, and ice cream, for example—increases a woman's risk of breast cancer threefold,"
Prevention Magazine's Giant Book of Health Facts (1991, p. 122)

If 1 in 9 women get breast cancer, does it mean if a women eats above diet, chances of breast cancer are 1 in 3?

Two problems:

- Don't know how study was conducted.
- Age is critical factor. The 1 in 9 is a lifetime risk, at least to age 85. ***Risk increases with age***.
- If study on young women, threefold increase is small.

Reported Risk versus Your Risk

"Older cars stolen more often than new ones"
Davis (CA) Enterprise, 15 April 1994, p. C3

Reported among the 20 most popular auto models stolen [in California] last year, 17 were at least 10 years old."

Many factors determine which cars stolen:

- Type of neighborhood.
- Locked garages.
- Cars not locked nor have alarms.

"If I were to buy a new car, would my chances of having it stolen increase or decrease over those of the car I own now?"
Article gives no information about that question.

12.4 Simpson's Paradox: The Missing Third Variable

- **Relationship appears to be in one direction if third variable is *not* considered and in other direction if it is.**
- **Can be dangerous to summarize information over groups.**

Example 7: Simpson's Paradox for Hospital Patients

Survival Rates for Standard and New Treatments

	Hospital A			Hospital B		
	Survive	Die	Total	Survive	Die	Total
Standard	5	95	100	500	500	1000
New	100	900	1000	95	5	100
Total	105	995	1100	595	505	1100

Risk Compared for Standard and New Treatments

	Hospital A	Hospital B
Risk of dying with the standard treatment	95/100 = 0.95	500/1000 = 0.50
Risk of dying with the new treatment	900/1000 = 0.90	5/100 = 0.05
Relative risk	0.95/0.90 = 1.06	0.50/0.05 = 10.0

Looks like ***new treatment is a success*** at both hospitals, especially at Hospital B.

Example 7: Simpson's Paradox for Hospital Patients

Estimating the Overall Reduction in Risk

	Survive	Die	Total	Risk of Death
Standard	505	595	1100	595/1100 = 0.54
New	195	905	1100	905/1100 = 0.82
Total	700	1500	2200	

What has gone wrong? With combined data it looks like the ***standard treatment is superior!*** Death rate for standard treatment is only 66% of what it is for the new treatment.

HOW?

More serious cases were treated at Hospital A (famous research hospital); more serious cases were also more likely to die, no matter what. ***And*** a higher proportion of patients at Hospital A received the new treatment.

Case Study 12.1: Assessing Discrimination in Hiring and Firing

Layoffs by Ethnic Group for Labor Department Employees

	Laid Off?			
Ethnic Group	Yes	No	Total	% Laid Off
African American	130	1382	1512	8.6
White	87	2813	2900	3.0
Total	217	4195	4412	

- Selection ratio of those laid off = 3.0/8.6 = 0.35
- Selection ratio of those retained = 91.4/97 = 0.94
- Discrepancy handled using Odds Ratio = $\frac{130 \times 2813}{1382 \times 87} = 3.04$

*Odds of being laid off compared with being retained are **three times higher** for African Americans than for whites.*

Source: Gastwirth and Greenhouse, 1995.

For Those Who Like Formulas

To represent the *observed numbers* in a 2×2 contingency table, we use the notation:

	Variable 2		
Variable 1	Yes	No	Total
Yes	a	b	$a + b$
No	c	d	$c + d$
Total	$a + c$	$b + d$	n

Relative Risk and Odds Ratio

Using the notation for the observed numbers, if variable 1 is the explanatory variable and variable 2 is the response variable, then we can compute

$$\text{relative risk} = \frac{a(c+d)}{c(a+b)}$$

$$\text{odds ratio} = \frac{ad}{bc}$$